BOTSWANA EXAMINATIONS COUNCIL
Botswana General Certificate of Secondary Education

MATHEMATICS **0563/03**

Paper 3 **October/November 2013**

2 hours 30 minutes

Additional Materials: Answer paper
Electronic calculator
Geometrical instruments
Graph Paper (2 sheets)
Mathematical tables (optional)

READ THESE INSTRUCTIONS FIRST

Write your answers on the separate answer paper provided.

Start each question on a fresh side of the page.

Write your centre number, candidate number and name on each sheet of answer paper you use.

Answer **all** questions.

All working must be clearly shown. The working should be done on the same sheet as the rest of the answer. Marks will be given for working which shows that you know how to solve the problem even if you get the answer wrong.

At the end of the examination, fasten all your work securely together using the string provided.

Do not use staples, paper clips, highlighters, glue or correction fluid.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 125.

Electronic calculators may be used.

If the degree of accuracy is not specified in the question and if the answer is not exact, the answer should be given to three significant figures. Answers in degrees should be given to one decimal place.

In any question where the value of π is required, use the value from your calculator or take π as 3.142.

This document consists of **9** printed pages and **3** blank pages.

Mathematical formulae for paper 3

Surface area and volume of solids

Name of solid	Total surface area	Volume
cone	$\pi r^2 + \pi rl$	$\frac{1}{3}\pi r^2 h$
pyramid		$\frac{1}{3}$ base area x height
sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Trigonometry

Sine Rule $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle $\quad = \frac{1}{2}ab\sin C$

Cosine Rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Statistics

$$\text{Variance} = \frac{\sum(x-\bar{x})^2}{n}, \qquad \frac{\sum f(x-\bar{x})^2}{\sum f}$$

$$\text{Standard deviation (SD)} = \sqrt{\text{Variance}} = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}, \qquad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

$$\text{or} \quad \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}, \qquad \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$$

1 A shopkeeper buys some stock items from a wholesaler.
He later sells each item at a profit of 40%.

(a) A pair of shoes was bought at P520 from the wholesaler.
For how much should the shopkeeper sell the pair of shoes? [3]

(b) The shopkeeper sold a T-shirt at P70.
For how much did the shopkeeper buy the T-shirt from the wholesaler? [3]

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2 The diagrams below show patterns made of matchsticks of the same length. The total number of sticks in Diagram 1 is 3 and the total number of sticks in Diagram 2 is 9.

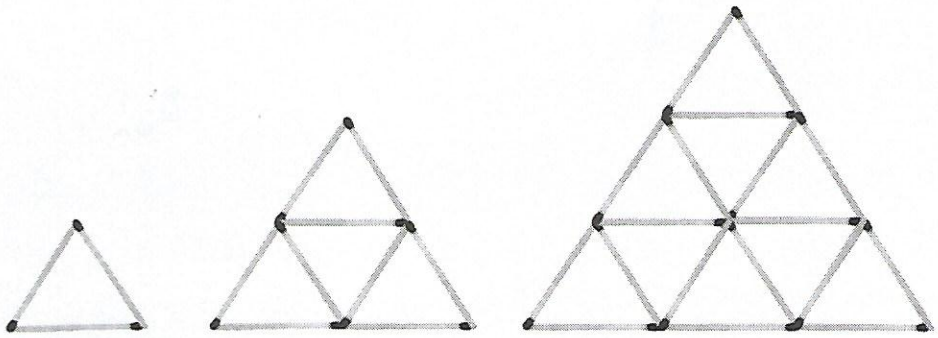

Diagram 1 Diagram 2 Diagram 3

The table below shows the total number of matchsticks used in a diagram.

Diagram	1	2	3	4	...	...	*n*
Total number of matchsticks	3	9			...	...	

(a) What is the total number of matchsticks in Diagram 3? [1]

(b) Find the total number of matchsticks in Diagram 4. [1]

(c) Express, in terms of n, the total number of matchsticks in Diagram n. [3]

(d) The total number of matchsticks in a diagram is 165.
What is the diagram number? [2]

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3 A factory packages 210 000 kg of tea every month.

(a) Express in standard form the mass of tea packaged each month. [1]

(b) The tea was packaged in packets of 50 tea bags each.
Each tea bag contains 2.5 grams of tea.

Calculate

(i) the mass of tea in each packet, [2]

(ii) the number of packets of tea that were packaged by the factory in a month.
Give the answer in standard form. [3]

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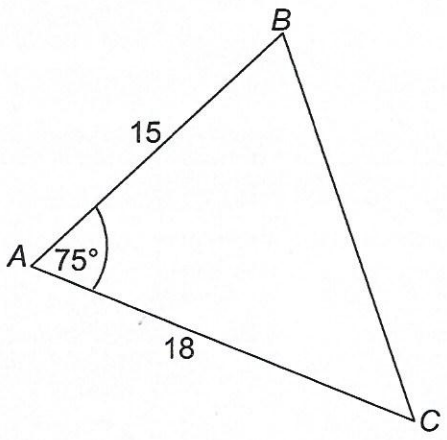
4 A tile ABC is in the form of a triangle as shown below.
$AB = 15\,\text{cm}$ and $AC = 18\,\text{cm}$. The size of $B\hat{A}C = 75°$.
All the measurements are given correct to the nearest unit.



(a) State the upper bound of

(i) AB, [1]

(ii) $B\hat{A}C$. [1]

(b) Calculate the upper bound for the maximum possible area that can be covered by the tile, giving your answer correct to one decimal place. [3]

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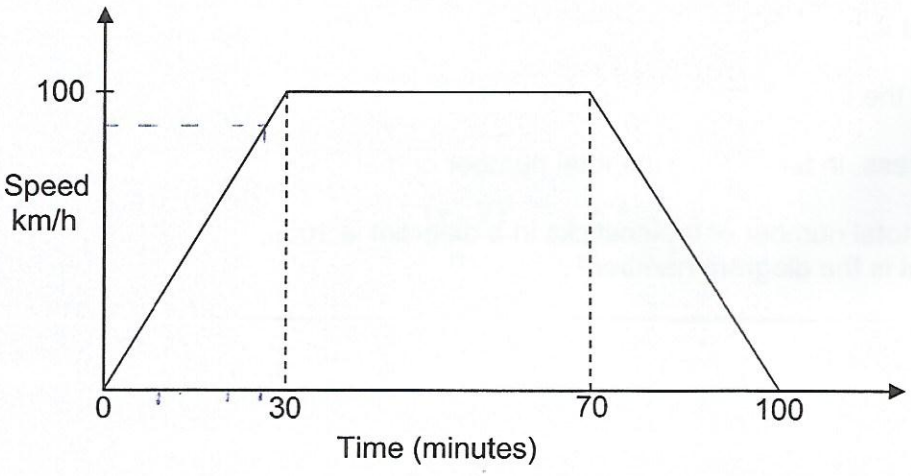
5 The diagram below shows a speed-time graph of a car.
The car accelerates to a speed of 100 km/h in 30 minutes.
It then travels at a constant speed for 40 minutes before stopping after a further 30 minutes.



Calculate

(a) the acceleration of the car between 0 and 30 minutes, [2]

(b) the speed of the car after 25 minutes, [2]

(c) the total distance travelled in 100 minutes. [3]

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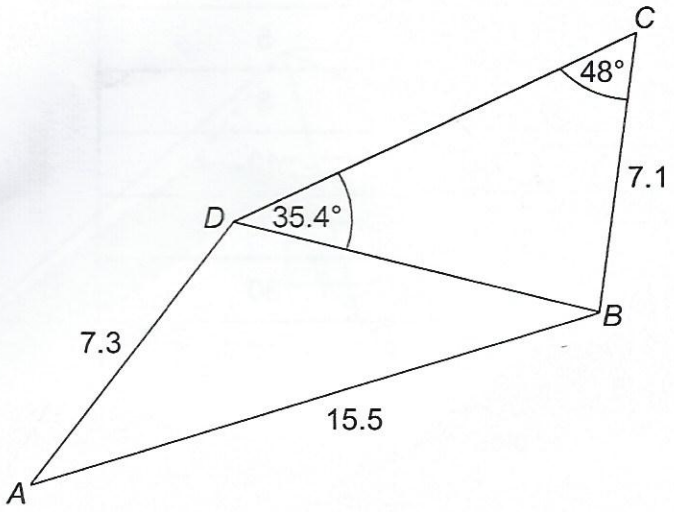
6 The diagram below shows two triangular plots BCD and ABD on a level ground. BD is a common boundary of the plots. $AD = 7.3$ m, $AB = 15.5$ m and $CB = 7.1$ m. The size of $B\hat{D}C = 35.4°$ and the size of $B\hat{C}D = 48°$.



(a) Show that the common boundary, BD, is 9.11 m long, correct to 3 significant figures. [3]

(b) Calculate the size of $A\hat{D}B$. [4]

(c) A vertical sign post, QT, is erected at Q along the boundary CD. QB is the shortest distance from B to CD.

(i) Calculate QB, correct to 2 decimal places. [2]

(ii) The height of the sign post is 5.5 m. Calculate the angle of elevation of T from B. [2]

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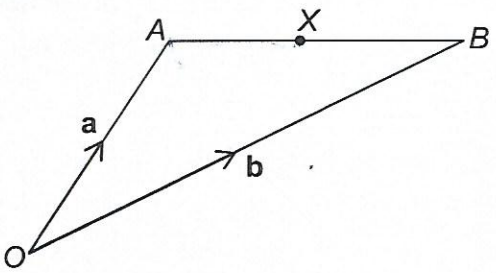
7 In the diagram below $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$. X is the midpoint of AB.



Express in terms of **a** and/or **b**

(a) $\overrightarrow{AB}$, [2]

(b) $\overrightarrow{AX}$, [1]

(c) $\overrightarrow{OX}$. [2]

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8 The table below shows the number of apples in each of 30 bags of apples in a shop.

Number of apples	*Number of bags*
15	5
16	8
17	10
18	7
Total	30

(a) Calculate the range. [2]

(b) What is the median number of apples in a bag? [2]

(c) Calculate

(i) the mean, [3]

(ii) the variance, [3]

(iii) the standard deviation. [2]

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9 A bag contains 6 black balls and 4 white balls of the same size.
One ball is selected at random from the bag.

(a) What is the probability, in the lowest terms, that the ball selected is white? [2]

(b) Two balls are selected from the bag one after the other.

What is the probability, in the lowest terms, that the balls selected

(i) are both black, [2]

(ii) are of different colours? [3]

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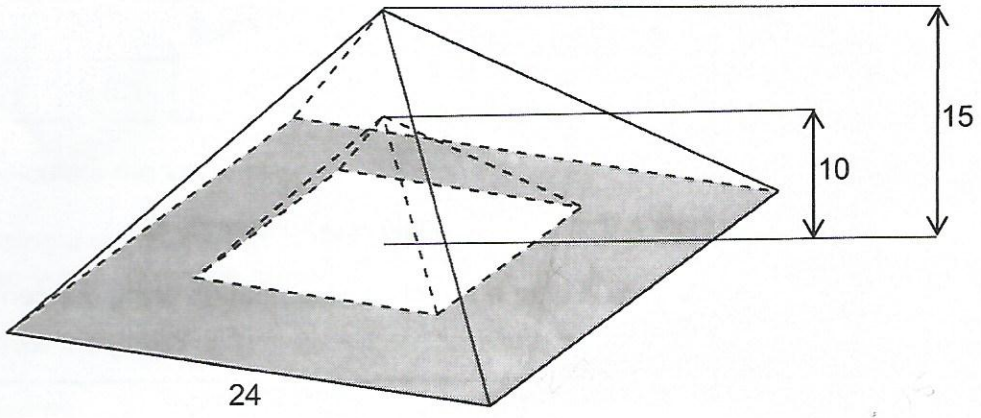
10 The diagram below shows a concrete cap in the form of a square based pyramid of side length 24 cm and height 15 cm. The cap is hollow with an inner similar pyramid of height 10 cm. The shaded area shows the base of the cap.



(a) Show that the side length of the inner pyramid is 16 cm. [2]

(b) Calculate the volume of the inner pyramid. [2]

(c) Show that the volume of the concrete that was used to make the cap is 2030 cm^3, correct to 3 significant figures. [3]

(d) The density of the concrete is 1.2 g/cm^3.

Calculate the mass of the concrete that was used to make the cap. [2]

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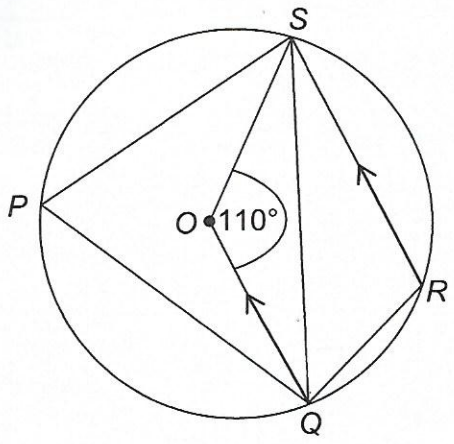
11 The points *P*, *Q*, *R* and *S* are on the circumference of the circle of centre *O* as shown below. *QO* is parallel to *RS*, and the size of $S\hat{O}Q = 110°$.



State with reasons the size of

(a) $Q\hat{P}S$, [2]

(b) $Q\hat{R}S$, [2]

(c) $S\hat{Q}R$. [2]

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12 Answer the whole of this question on sheet of graph paper.

Using a scale of 1cm to represent 1 unit on each axis, draw a pair of axes for $-5 \leq x \leq 7$ and $-5 \leq y \leq 10$.

(a) Draw and label

(i) triangle A with vertices (1, 5), (3, 5) and (3, 2), [1]

(ii) triangle B with vertices (3, 8), (7, 8) and (7, 2). [1]

(b) Describe fully the transformation that maps triangle A onto triangle B. [3]

(c) Triangle D is the image of triangle A after a 90° clockwise rotation using the centre (0, 0). Draw and label the image D. [2]

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13 The cost of a pencil is Px and the cost of a pen is Py.

(a) Thabo bought 5 pencils.
Express, in terms of x and/or y, the cost of the 5 pencils. [1]

(b) Maria bought 4 pens.
Express, in terms of x and/or y, the cost of the 4 pens. [1]

(c) The total cost of the 5 pencils and the 4 pens is P18.
Write down an equation in terms of x and y to show this information. [2]

(d) Bathusi bought 10 pencils and 6 pens at P29.
Write down an equation in terms of x and y to show this information. [2]

(e) Solve simultaneously the equations in **(c)** and in **(d)**. [3]

(f) Calculate the cost of 3 pencils and 2 pens. [2]

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14 Answer the whole of this question in a sheet of graph paper.

The table below shows the values of x and the corresponding values of y for the equation $y = x^3 - 3x^2 + 4$.

x	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3	4
y	–6.1	0	3.1	4	3.4	2	0.6	0	0.9	4	p

(a) Calculate the value of p. [1]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw the graph of $y = x^3 - 3x^2 + 4$ for $-2 \le x \le 4$. [4]

(c) Draw the graph of $3y = 2x + 4$ on the same pair of axes. [2]

(d) Use the graphs in **(b)** and in **(c)** to estimate the values of x that satisfy the equations $y = x^3 - 3x^2 + 4$ and $3y = 2x + 4$. [3]

(e) By drawing a tangent, estimate the gradient of the curve when $x = 2.5$. [3]

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15 Ben and Tumi took part in a 45 km bicycle race.

(a) Ben cycled at a constant speed of v km/h for the entire race.
Express the time taken by Ben in terms of v. [1]

(b) Tumi cycled at a constant speed of 3 km/h faster than Ben.

Express

(i) Tumi's constant speed in terms of v, [1]

(ii) the time taken by Tumi in terms of v. [1]

(c) The time taken by Tumi is $\frac{1}{4}$ of an hour less than the time taken by Ben.
Form an equation in v and show that it reduces to $v^2 + 3v - 540 = 0$. [3]

(d) (i) Solve the equation $v^2 + 3v - 540 = 0$, giving the answers correct to one decimal place. [5]

(ii) Calculate Tumi's speed. [2]
