

Mathematical formulae for paper 3

Surface area and volume of solids

Name of solid	Total surface area	Volume
cone	$\pi r^2 + \pi r l$	$\frac{1}{3}\pi r^2 h$
pyramid		$\frac{1}{3}$ base area \times height
sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Trigonometry

Sine Rule $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle $= \frac{1}{2}ab \sin C$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Statistics

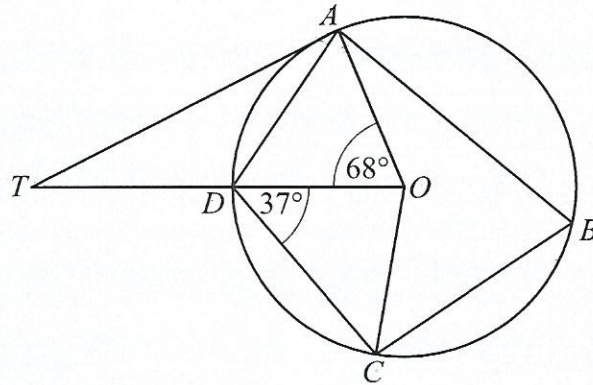
$$\text{Variance} = \frac{\sum(x - \bar{x})^2}{n}, \quad \frac{\sum f(x - \bar{x})^2}{\sum f}$$

$$\text{Standard deviation (SD)} = \sqrt{\text{Variance}} = \sqrt{\frac{\sum(x - \bar{x})^2}{n}}, \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

or $\sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}, \quad \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$

- 1 A country has 3 582 600 people who qualify to vote. During the voter registration exercise, 2 300 000 people registered.
- (a) (i) Calculate the number of people who qualify to vote that did not register. [2]
- (ii) Express the number of people who did not register as a percentage of those who qualify to vote. [2]
- (b) On the election day 80% of registered voters voted.
- Calculate the number of registered voters who voted, giving the answer in standard form. [3]
- (c) The 3 582 600 people who qualify to vote is 42% of the total population of the country.
- Calculate the total population of the country. [3]
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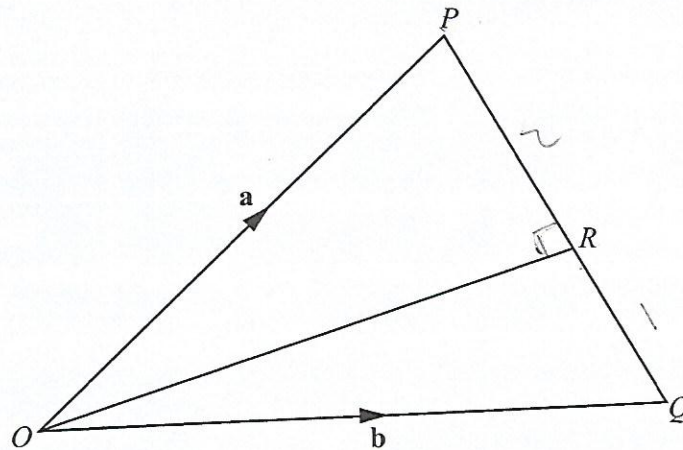
- 2 The diagram below shows a circle with centre O .
 The points A, B, C and D are on the circumference of the circle.
 TA is a tangent to the circle and OT is a straight line through D .
 $\hat{AOD} = 68^\circ$ and $\hat{ODC} = 37^\circ$.



State, with reasons, the size of

- (a) \hat{ATO} , [2]
- (b) \hat{DAT} , [2]
- (c) \hat{ABC} . [2]
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- 3 The diagram below shows triangle OPQ . The position vectors of P and Q are \mathbf{a} and \mathbf{b} respectively. R is a point on PQ such that $PR:RQ = 2:1$.



- (a) Write down, in terms of \mathbf{a} and/or \mathbf{b} , an expression for
- (i) \vec{PQ} , [2]
- (ii) \vec{OR} . [3]
- (b) Find the ratio of the area of triangle OPR to the area of triangle OPQ . [2]
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- 4 Kago spends P510 on meat in the month of June.

- (a) The meat costs Px per kg.

Express, in terms of x , the amount of meat, in kg, that Kago buys in June. [1]

- (b) In July, the cost per kg of meat increased by P2.

Express, in terms of x ,

- (i) the new cost of meat per kg, [1]
- (ii) the amount of meat, in kg, that Kago will now buy with the same amount of money. [1]

- (c) The amount of meat he can now buy is 1.5 kg less than the amount of meat he could have bought in June.

- (i) Form an equation in x for this information and show that it reduces to $x^2 + 2x - 680 = 0$. [3]
- (ii) Solve the equation $x^2 + 2x - 680 = 0$, giving your answers correct to 2 decimal places. [5]

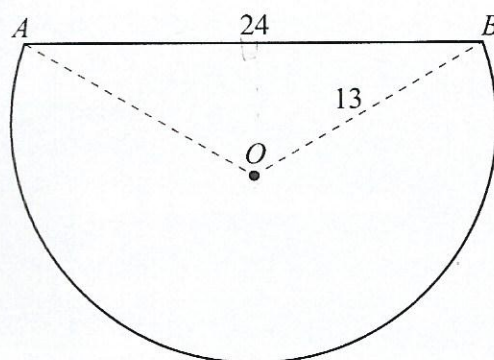
- (d) Hence, or otherwise, calculate the cost per kg of meat in July. [2]
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- 5 The table below shows the ages of 18 members of a chess club, 10 boys and 8 girls.

Age (years)	Boys	Girls
17	2	1
18	3	2
19	4	2
20	1	3
Total	10	8

- (a) A member is chosen at random from the club to attend a meeting.
What is the probability that the member is a girl? [1]
- (b) The club selects two members to attend an under-20 workshop.
- (i) How many members do **not** qualify to attend the workshop? [1]
- (ii) Calculate the probability that two girls are selected to attend the workshop. [3]
- (iii) Two girls are out to attend the workshop.
In their absence, two of the other 16 members are selected to compete in a tournament.
Calculate the probability that a boy and a girl are selected to compete in the tournament. [3]

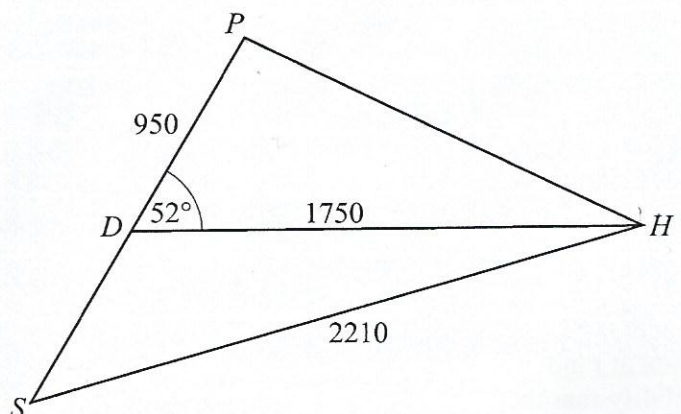
- 6 The diagram below shows a cross-section of a wooden pole.
The cross-section is a segment of a circle centre O , radius 13 cm and chord $AB = 24$ cm.
The pole is 150 cm long.



- (a) Show that the size of $\hat{AOB} = 134.8^\circ$, correct to 1 decimal place. [2]
- (b) Calculate
- (i) the length of the arc AB , [3]
- (ii) the area of the cross-section of the pole. [3]
- (c) The whole pole is to be painted.
Calculate the total area to be painted. [4]

- 7 The diagram below shows the positions of a police station, P , a shop, S , a hospital, H and a district office, D .

$DP = 950$ m, $DH = 1750$ m, $SH = 2210$ m and $\widehat{HDP} = 52^\circ$.



- (a) Calculate

(i) the distance, HP , from the hospital to the police station, [4]

(ii) the size of \widehat{DSH} . [3]

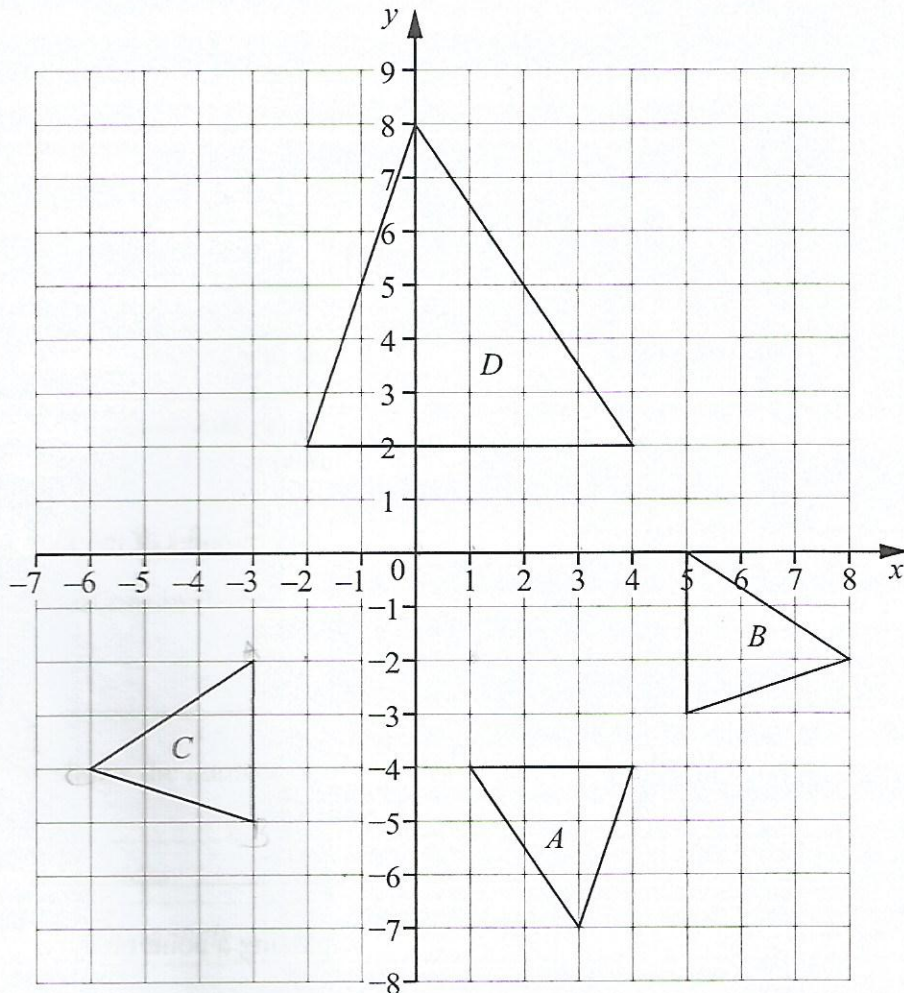
- (b) A point X is along DH such that the distance of the police station, P , from DH is shortest.

Calculate PX . [2]

- (c) A helicopter hovers directly above the police station, P , at a height of 575 m.

Calculate the angle of elevation of the helicopter from the district office, D . [2]

- 8 The diagram below shows the triangles A , B , C and D .



- (a) Describe fully the single transformation that maps triangle A onto triangle

(i) B , [2]

(ii) C . [3]

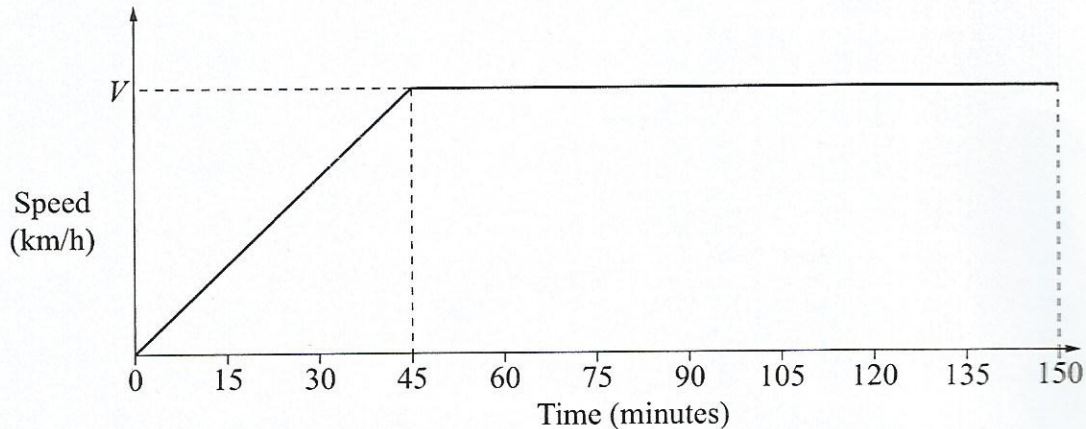
- (b) Triangle D is mapped onto triangle A by an enlargement.

Find

(i) the scale factor of enlargement, [1]

(ii) the centre of enlargement. [1]

- 9 The diagram below shows a speed-time graph for the first 150 minutes of a donkey-cart journey. The greatest speed reached by the donkey-cart is V km/h.



- (a) The donkey-cart travelled a distance of 7 km in the first 45 minutes of the journey.
Calculate the value of V , in km/h. [2]
- (b) Calculate
- (i) the acceleration in the first 45 minutes, in km/h^2 . [2]
- (ii) the total distance travelled in the 150 minutes. [3]

- 10 The table below shows the speeds, in km/h, of 60 vehicles passing a point on a highway.

Speed (km/h)	$0 < s \leq 40$	$40 < s \leq 60$	$60 < s \leq 80$	$80 < s \leq 100$	$100 < s \leq 140$
Number of vehicles	9	14	18	12	7

- (a) Find the mid-value of the class $0 < s \leq 40$. [1]
- (b) Calculate
- (i) the mean speed of the vehicles, [3]
- (ii) the variance, [4]
- (iii) the standard deviation. [1]
- (c) It was found that 20% of the vehicles were over speeding.
Without drawing a cumulative frequency curve, calculate an estimate of the speed limit. [3]

11 A small cup has a volume of x ml and a large cup has a volume of y ml.

Tebo pours 3 small cups and 4 large cups of milk into a bucket.

(a) Express, in terms of x and/or y , the total volume of milk poured into the bucket using

(i) the small cup, [1]

(ii) both the small cup and the large cup. [2]

(b) Tebo poured a total of 2375 ml of milk into the bucket.

Form an equation in x and y to show this information. [1]

(c) Kapande pours a total volume of 2750 ml of milk into the bucket using 2 small cups and 5 large cups.

Form an equation in x and y to show this information. [2]

(d) Using the equations in (b) and (c), find the volume of the small cup and the volume of the large cup. [3]

12 The table below shows the number of diagonals in polygons.

<i>Number of sides</i>	3	4	5	6	7	8	9	n
<i>Number of diagonals</i>	0	2	5	9	14				

A 3-sided polygon has 0 diagonals, a 4-sided polygon has 2 diagonals, a 5-sided polygon has 5 diagonals, and so on.

(a) How many diagonals are there in

(i) an 8-sided polygon, [1]

(ii) a 9-sided polygon. [1]

(b) Find the general formula for the number of diagonals in an n -sided polygon. [2]

(c) Calculate the number of sides of a polygon that has 104 diagonals. [3]

13 Answer the whole of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y , correct to 1 decimal place, for the equation

$$y = 5 - \frac{3}{x^2}.$$

x	0.5	0.75	1	1.25	1.5	1.75	2	3	4
y	-7	-0.3	2	3.1	3.7	4.0	4.3	4.7	a

- (a) State the value of x which **cannot** be used when calculating values of y . [1]
- (b) Calculate the value of a . [1]
- (c) Use a scale of 2 cm to represent 1 unit on the x -axis, for $-4 \leq x \leq 4$, and 1 cm to represent 1 unit on the y -axis, for $-8 \leq y \leq 6$, to draw the graph of $y = 5 - \frac{3}{x^2}$. [3]
- (d) The line of symmetry of the graph of $y = 5 - \frac{3}{x^2}$ is the y -axis.

The table below shows some values of x and the corresponding values of y , for the graph after a reflection in the y -axis.

x	-0.5	-0.75	-1	-1.25	-1.5	-1.75	-2	q	-4
y	p	-0.3	2	3.1	3.7	4.0	4.3	4.7	a

- (i) Find the values of p and q . [1]
- (ii) Draw, on the same axes, the graph of $y = 5 - \frac{3}{x^2}$ after a reflection in the y -axis for values of x from -0.5 to -4 . [3]
- (e) (i) On the same axes, draw the graph of $y = -x - 1$. [2]
- (ii) Hence, or otherwise, find the values of x that satisfy the equation $5 - \frac{3}{x^2} = -x - 1$. [2]
- (f) By drawing a tangent, estimate the gradient of $y = 5 - \frac{3}{x^2}$ when $x = 1\frac{1}{2}$. [3]
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