

Mathematical formulae for paper 3

Surface area and volume of solids

Name of solid	Total surface area	Volume
cone	$\pi r^2 + \pi r l$	$\frac{1}{3} \pi r^2 h$
pyramid		$\frac{1}{3}$ base area \times height
sphere	$4\pi r^2$	$\frac{4}{3} \pi r^3$

Trigonometry

Sine Rule $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle $= \frac{1}{2} ab \sin C$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Statistics

$$\text{Variance} = \frac{\sum (x - \bar{x})^2}{n}, \quad \frac{\sum f(x - \bar{x})^2}{\sum f}$$

$$\text{Standard deviation (SD)} = \sqrt{\text{Variance}} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}, \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

or $\sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}, \quad \sqrt{\frac{\sum fx^2}{\sum f} - (\bar{x})^2}$

- 1 (a) A customer paid P280 as lay-bye for a generator.
This is $\frac{1}{3}$ of the total cost of the generator.
Calculate the balance to be paid. [2]
- (b) The diagram below shows a computer tower with the original price and the price after a discount.

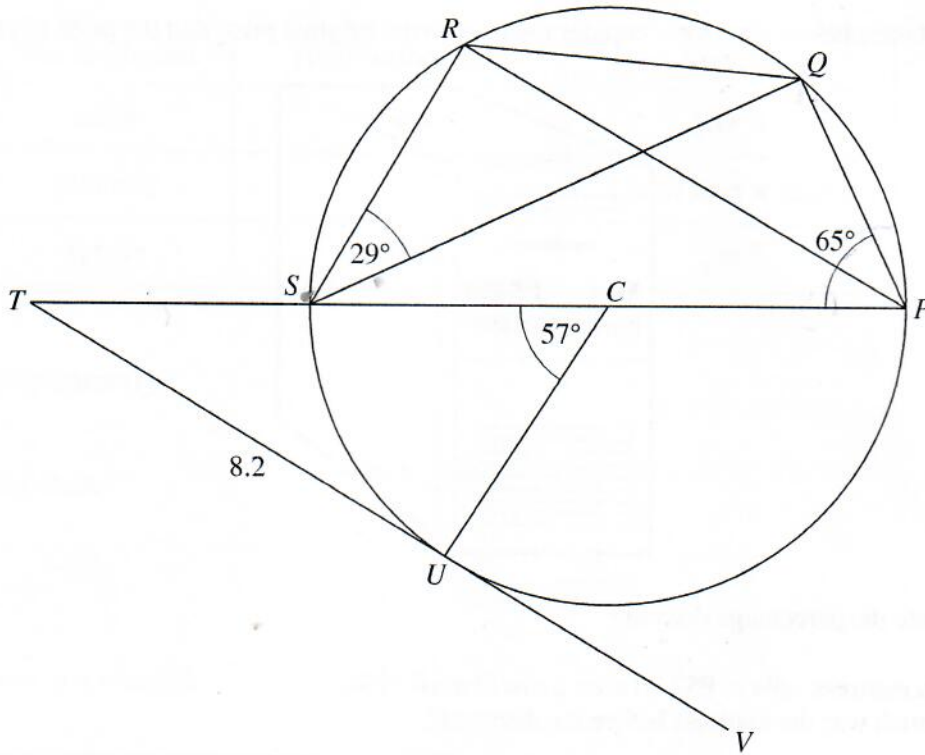


Calculate the percentage discount. [2]

- (c) A foam mattress sells at P52.80 after a discount of 12%.
How much was the mattress before the discount? [3]

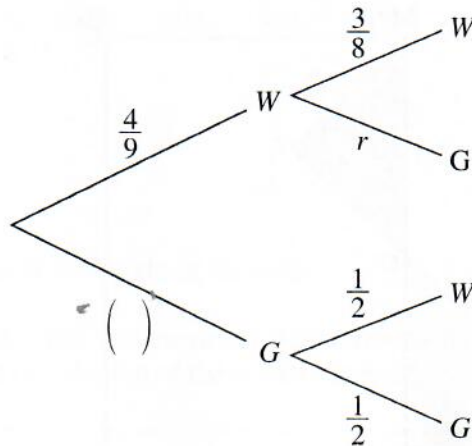
- 2 In a country of 4.9×10^7 people, a town has a population of 9.8×10^4 .
- (a) Express the population of the town as a fraction of the population of the country. Give your answer in standard form. [2]
- (b) During an election, only half of the population of the country voted.
How many people voted? Give the answer in standard form. [2]
- (c) The population of the town increases by an average of 5% annually.
Giving the answer in standard form, calculate the expected population after
- (i) one year, [3]
- (ii) two years. [2]

- 3 In the diagram, the points P, Q, R, S and U lie on the circumference of a circle with centre C . TUV touches the circle at U .
 $\hat{R}SQ = 29^\circ$, $\hat{T}CU = 57^\circ$, $\hat{S}PQ = 65^\circ$ and $TU = 8.2$ cm.



- (a) What is
- the geometrical name for line TV ? [1]
 - the size of $\hat{T}UC$? [1]
- (b) Calculate the radius, CU , of the circle. [2]
- (c) State, with a reason, the size of
- $\hat{R}PQ$, [2]
 - $\hat{S}RQ$. [2]

- 4 A bag contains 9 identically-sized marbles: 4 white and 5 green.
Without replacing, Thabo takes 2 marbles at random, one after the other from the bag.
The tree diagram below shows all the possible ways in which he may pick the marbles.

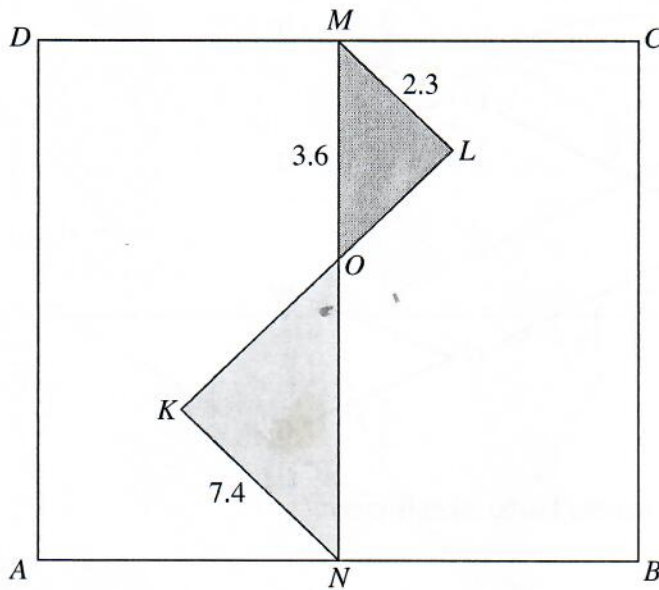


- (a) Find the probability that the first marble Thabo takes is green. [1]
- (b) Calculate the value of r . [1]
- (c) Calculate the probability that the marbles taken are
- (i) both green, [2]
- (ii) of different colours. [2]

- 5 Answer the whole of this question on a sheet of graph paper.

- (a) Using a scale of 1 cm to represent 1 unit on each axis, draw x - and y -axes for $-10 \leq x \leq 4$ and $-6 \leq y \leq 12$.
Draw triangle ABC with vertices $A(-5, 4)$, $B(-3, 2)$ and $C(-1, 2)$. [1]
- (b) Triangle DEF has vertices $D(-7, 2)$, $E(-5, 0)$ and $F(-5, -2)$.
- (i) Draw triangle DEF . [1]
- (ii) A reflection maps triangle ABC onto triangle DEF .
Draw the line of reflection. [1]
- (c) Triangle GHI is the image of triangle ABC under an enlargement, centre $(-2, 5)$ and scale factor -2 .
Draw triangle GHI . [3]
- (d) Triangle JKL has vertices $J(4, 3)$, $K(2, 1)$ and $L(2, -1)$.
- (i) Draw triangle JKL . [1]
- (ii) Describe fully the single transformation that maps triangle ABC onto triangle JKL . [3]

- 6 The diagram below shows a square wall tile, $ABCD$.
The tile has a print in the form of two similar triangles OLM and OKN as shown.
 $LM = 2.3$ cm, $OM = 3.6$ cm and $KN = 7.4$ cm.



NOT TO SCALE

- (a) Calculate the length of ON . [2]
- (b) The area of triangle OKN is 27.5 cm^2 .
Calculate the area of triangle OLM . [3]
- (c) The points N and M are mid-points of AB and DC respectively.
Calculate the area of the square wall tile. [2]
- (d) A rectangular wall with length 340 cm and height 200 cm is to be tiled using the square wall tiles.
- (i) How many tiles will be needed to go along the length of the wall? [1]
- (ii) How many tiles will be needed to cover the wall? [2]

7 Answer the part (b) of this question on a sheet of graph paper.

- (a) The distribution below shows the height of a wall, in mm, measured at 4 different positions.

302, 308, 305, 306.

Calculate

- (i) the mean, [2]
 (ii) the variance, [3]
 (iii) the standard deviation of the height of the wall. [1]
- (b) The thickness, in mm, of the wall was measured at 39 different positions along the wall. The table below shows the distribution of the results obtained.

Thickness (mm)	Number of positions
231–235	3
236–240	9
241–250	15
251–265	12

- (i) Calculate an estimate of the median. [3]
 (ii) Draw a histogram showing the distribution of the thickness of the wall at different positions. [3]
 (iii) Find the modal class of the distribution. [1]

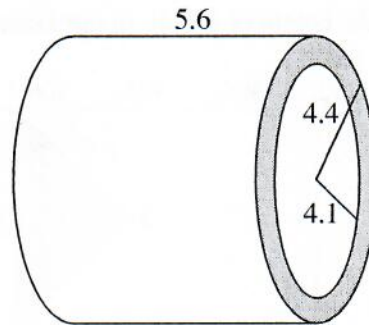
8 Answer the whole of this question on a sheet of graph paper.

The table below shows some values of x and the corresponding values of y for the equation $y = x^2 + x - 3$.

x	-3	-2	$-1\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
y	3	-1	-2.25	-3	-3.25	-3	-2.25	-1	3	p

- (a) Calculate the value of p . [1]
 (b) Using a scale of 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the y -axis, draw the graph of $y = x^2 + x - 3$ for $-3 \leq x \leq 3$. [3]
 (c) Use the graph to find the values of x such that $x^2 + x - 3 = 0$. [2]
 (d) On the same axes, draw the graph of $y = \frac{2}{3}x + 1$. [1]
 (e) Write down the values of x such that $x^2 + x - 3 = \frac{2}{3}x + 1$. [2]
 (f) By drawing a tangent estimate the gradient of $y = x^2 + x - 3$ at $x = \frac{1}{2}$. [3]

- 9 A cylindrical ring has length 5.6 cm, inner radius 4.1 cm and outer radius 4.4 cm, as shown in the diagram.

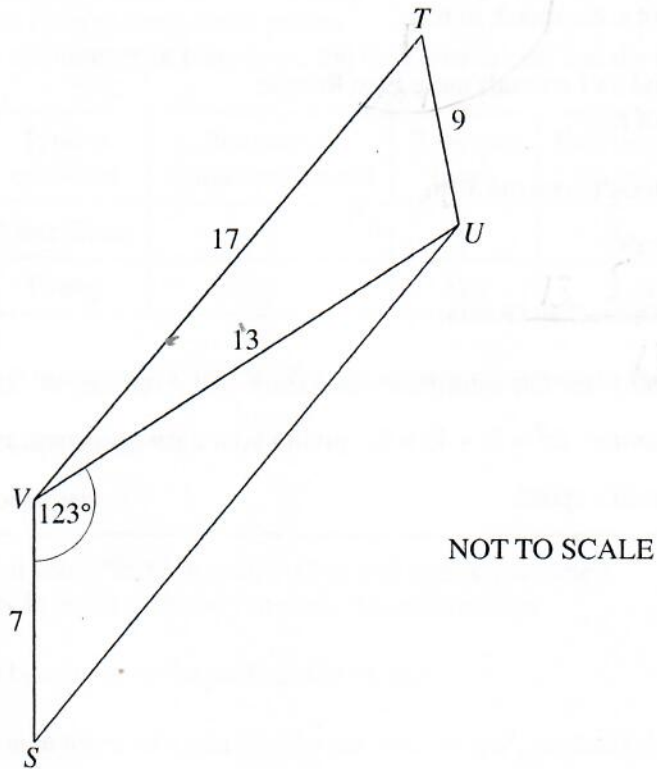


- (a) The ring is rolled on a flat table to make a complete revolution.

Calculate

- (i) the distance covered in one revolution, [2]
- (ii) the area of the cylindrical surface that was in contact with the flat table. [2]
- (b) The ring is made of steel alloy.
Calculate the volume of the steel alloy used to make the ring. [2]
- (c) The density of the steel alloy is 8 g/cm^3 .
Calculate the mass of the ring. [2]
- (d) The ring is to be covered completely in oil.
Calculate the total surface area to be covered. [4]

- 10 The diagram shows two adjacent triangular plots, TUV and UVS .
 $TV = 17\text{ m}$, $VU = 13\text{ m}$, $TU = 9\text{ m}$, $VS = 7\text{ m}$ and $\hat{SVU} = 123^\circ$.



- (a) Calculate \hat{TUV} . [4]
- (b) A flag pole, TW , of height 4 m, is to be erected at T .
 Calculate the angle of depression of V from W . [3]
- (c) The plot UVS is to be covered with lawn.
- (i) Calculate the area to be covered. [2]
- (ii) Lawn is sold at P4.75 per square metre.
 Calculate the amount of money needed to buy the lawn. [2]

11 Rombe and Gorata each ran a distance of 30 m.

- (a) Rombe took t seconds.
Express, in terms of t , his speed, in m/s. [1]
- (b) Gorata took one and half seconds more than Rombe.
Express, in terms of t ,
- (i) the time she took to run the 30 m, [1]
- (ii) her speed in m/s. [1]
- (c) Rombe was 2 m/s faster than Gorata.
- (i) Write an equation for this information and show that it reduces to $2t^2 + 3t - 45 = 0$. [3]
- (ii) Solve the equation $2t^2 + 3t - 45 = 0$, giving your answers correct to 2 decimal places. [5]
- (iii) Calculate Gorata's speed. [2]
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12 Answer the whole of this question on a sheet of graph paper.

Mrs Molemi buys two types of containers, those made of fibre glass and those made of plastic. The containers are in the form of rectangular prisms. The table below shows the number of containers, the base area in cm^2 and the unit price in Pula.

Type of container	Number of containers bought	Base area (cm^2)	Unit price (Pula)
Fibre Glass	x	98	8
Plastic	y	126	3

- (a) Write an expression in terms of x and/or y for the amount of money, in Pula, used to buy
- (i) the fibre glass containers, [1]
 - (ii) the plastic containers. [1]
- (b) Mrs Molemi uses at least P36 to buy fibre glass and plastic containers. Write an inequality in terms of x and y to show this information. [1]
- (c) All the containers bought are to be packed side by side.
- Write an expression in terms of x and/or y for the area, in cm^2 , needed for
- (i) the fibre glass containers, [1]
 - (ii) the plastic containers. [1]
- The area available to pack the fibre glass and the plastic containers is at most 882 cm^2 .
- (iii) Write an inequality in terms of x and y for this information and show that it reduces to
- $$7x + 9y \leq 63. \quad [2]$$
- (d) More fibre glass containers than plastic containers must be bought.
- Write an inequality in terms of x and/or y to show this information. [1]
- (e) (i) Using a scale of 2 cm to represent 2 units on both axes show, by shading the unwanted regions, the set of points satisfying the three inequalities in parts (b), (c) and (d). [4]
- (ii) Calculate the smallest amount of money that can be used to buy 8 containers. [2]